LIBOR-in-Arrears Swaps

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LIBOR-in-Arrears Swaps

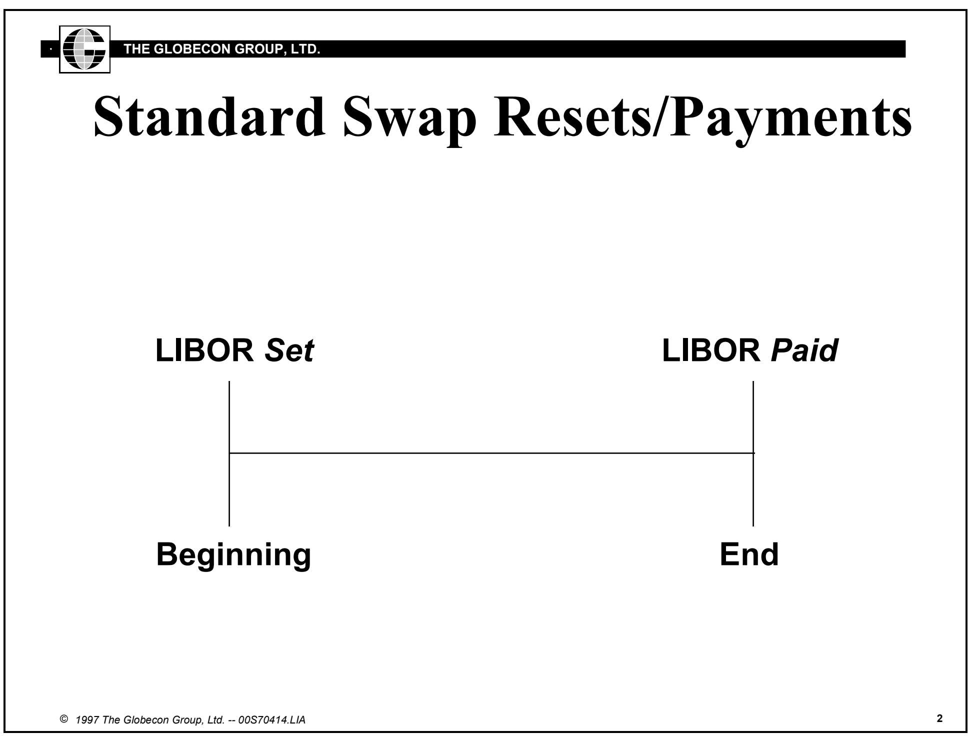
OVERVIEW

This module will cover LIBOR-in-Arrears swaps (LIA swaps), focusing on the application, pricing, and hedging of these swap structures. LIA swaps typically become interesting to clients whenever there is a steepness in the yield curve, either positive or negative. They are thus somewhat cyclical in their attractiveness, but are a fairly standard "product" for derivatives marketers.

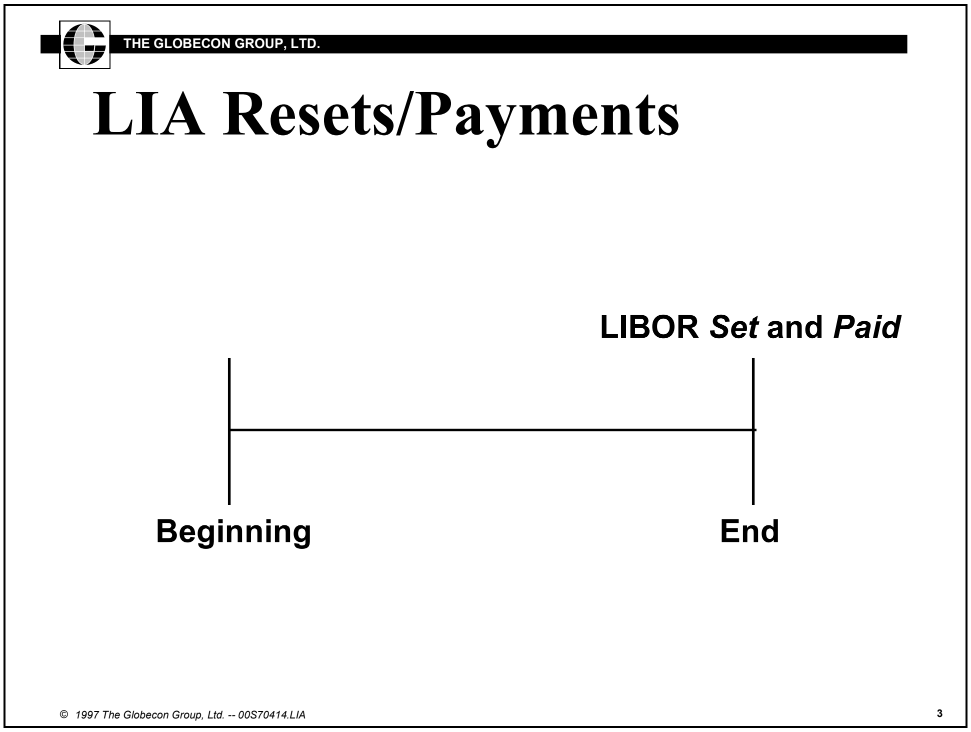
TAKING VIEWS ON YIELD CURVE STEEPNESS

Steep yield curve environments — both positive and negative — encourage paying or receiving LIBOR set at the end of the period instead of at the beginning.

The normal process of making payments in LIBOR **fixes** the rate at the beginning of the period and **pays** the rate at the end:



LIBOR-in-arrears payments change this pattern. LIBOR is **fixed at the end** of the period and **paid** at the end, two days later:



Normally, LIBOR is set using the rate for the same period beginning at the end of the relevant reset period.

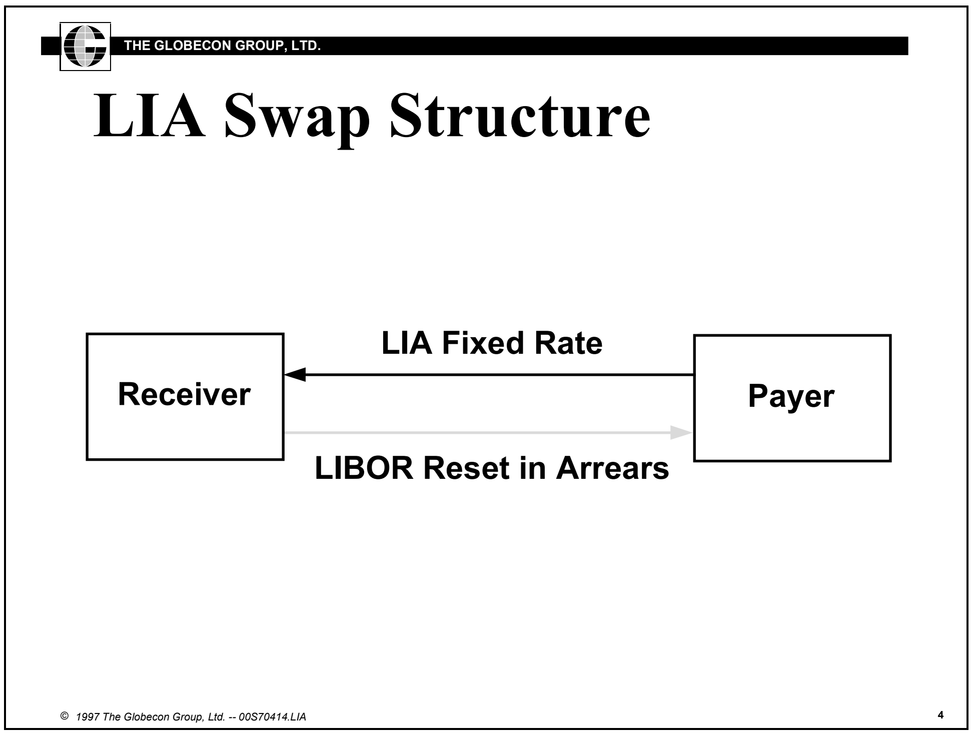
For example, if using six-month LIBOR resets, then the reset rate for the period just ending will be the market level of six-month LIBOR for the period just beginning.

This is not the only possible approach, however. One could choose to use the market rate for a longer or shorter LIBOR period, for example, or the five-year swap rate. This would produce a structure known as a “yield curve swap.”

LIBOR-IN-ARREARS SWAP STRUCTURE

Basic Swap Structure

The basic LIA swap is very much like a standard interest rate swap:



The LIA fixed rate is about equal to the rate on a forward starting swap of the same tenor beginning one LIBOR reset period forward.

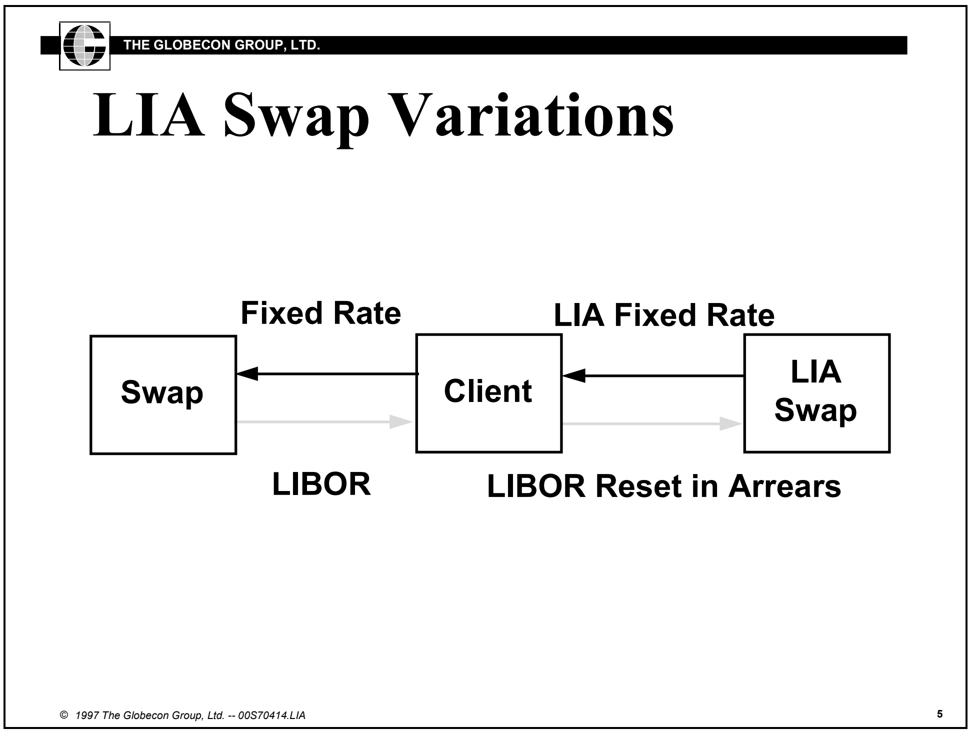
For an **upward-sloping** yield curve, the LIA fixed rate will be **higher** than the normal swap rate.

For a **downward-sloping** yield curve, the LIA fixed rate will be **lower** than the normal swap rate.

LIA SWAP VARIATIONS

Upward-Sloping Yield Curve

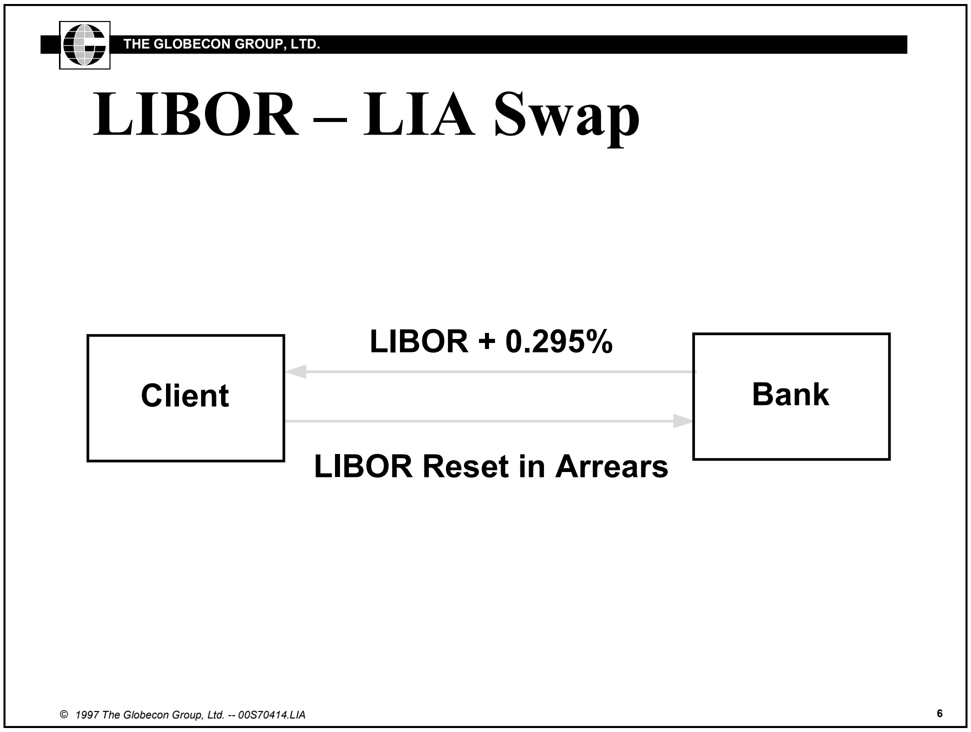
The LIA swap can be combined with a regular interest rate swap to exchange LIBOR in arrears against LIBOR set in the usual manner:



For an upward-sloping yield curve, the LIA fixed rate will be higher than the normal swap rate by a fixed amount.

For example, if the LIA swap rate is 5.95% and the normal swap rate is 5.655%, the client could receive the LIA spread of 0.295% each period.

This would produce the following position for the client:

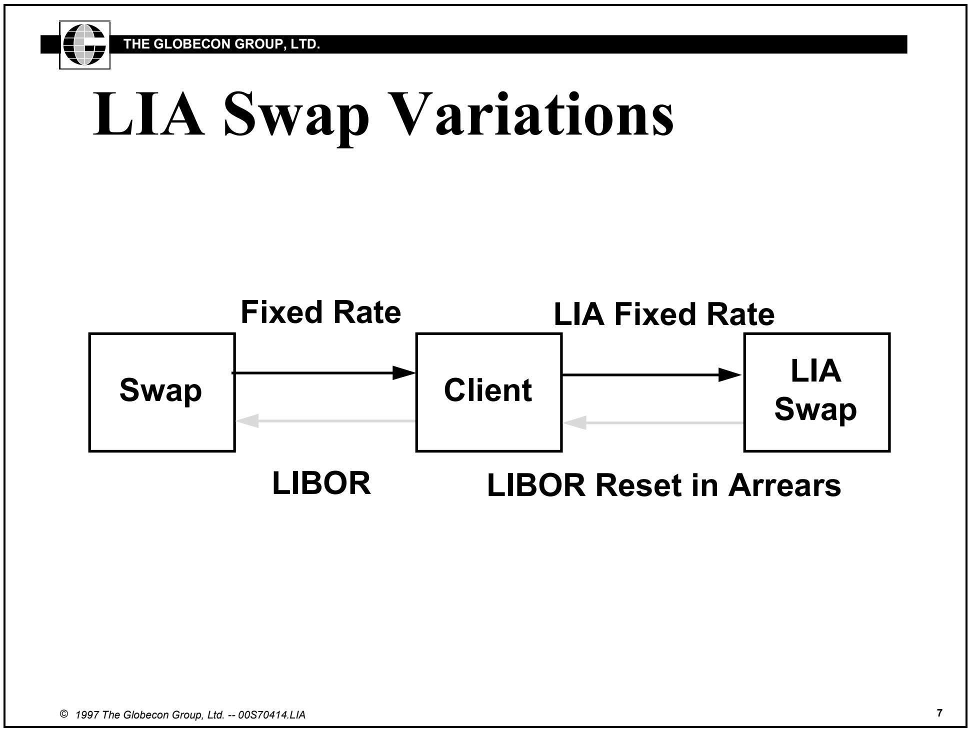


As long as LIBOR does not rise by more than 0.295% on average over each reset period, the client would come out ahead.

By taking this position, the client is stating the view that the forward curve "forecasts" LIBOR rising faster and sooner than the client believes will happen. The curve is too steeply upward sloping.

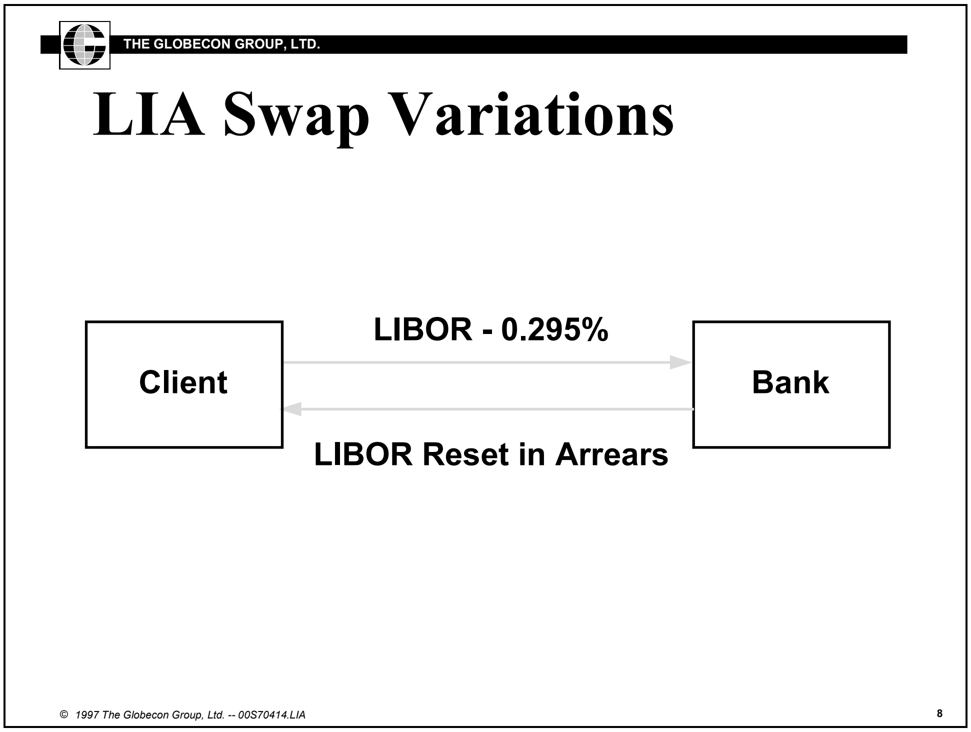
Downward-Sloping Yield Curve
For a downward-sloping yield curve, the LIA swap rate would be lower than the normal swap rate.

The client might prefer the following position:



If the LIA swap rate was 5.36% and the normal swap rate was 5.655%, the client would pay the LIA spread of –0.295% each period.

This would produce the following position for the client:



As long as LIBOR did not fall by more than 0.295% on average over each reset period, the client would come out ahead.

By taking this position, the client is stating the view that the forward curve “forecasts” LIBOR falling faster and sooner than the client believes will happen. The curve is too steeply downward sloping.

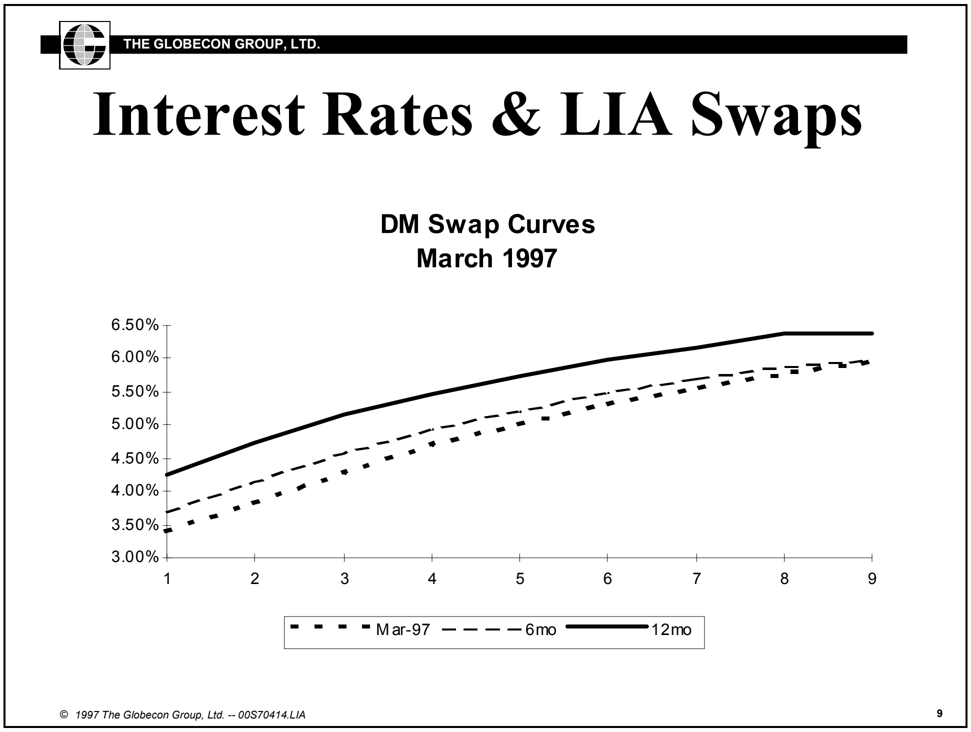
INTEREST RATES VIEWS AND LIA SWAPS

Following is an example of how to price LIA swaps. In March 1997 the following Deutschemark swap curves were available from the market:

Date	Days	Swaps	PV Factors	DM FRAs	Forward Swaps	
3/17/97			1.000000		**6 Months**	**12 months**
9/17/97	184		0.983564	3.3422%		
3/17/98	181	3.3867%	0.966803	3.4674%		
9/17/98	184		0.947657	4.0405%	3.7139%	
3/17/99	181	3.8350%	0.927359	4.3777%		4.2533%
9/17/99	184		0.904925	4.9582%	4.1516%	
3/17/00	182	4.2750%	0.881347	5.3505%		4.7247%
9/17/00	184		0.856352	5.8375%	4.5834%	
3/17/01	181	4.6950%	0.830689	6.1788%		5.1570%
9/17/01	184		0.805421	6.2744%	4.9399%	
3/17/02	181	5.0150%	0.780031	6.5101%		5.4621%
9/17/02	184		0.754275	6.8292%	5.2275%	
3/17/03	181	5.3050%	0.728655	7.0321%		5.7411%
9/17/03	184		0.703044	7.2858%	5.4844%	
3/17/04	182	5.5650%	0.677646	7.4961%		5.9920%
9/17/04	184		0.653725	7.3182%	5.6942%	
3/17/05	181	5.7550%	0.630363	7.4125%		6.1663%
9/17/05	184		0.605947	8.0585%	5.8792%	
3/17/06	181	5.9650%	0.582149	8.1759%		6.3703%
9/17/06	184		0.564307	6.3237%	5.9851%	
3/17/07	181	5.9950%	0.547241	6.2372%		6.3710%

- Swaps are quoted on a 30/360 basis.
- The three-year swap rate is 4.275%.
- The three-year swap rate six months forward is 4.583%.
- The three-year swap rate one year forward is 5.157%.

Graphing these rates illustrates them more clearly:



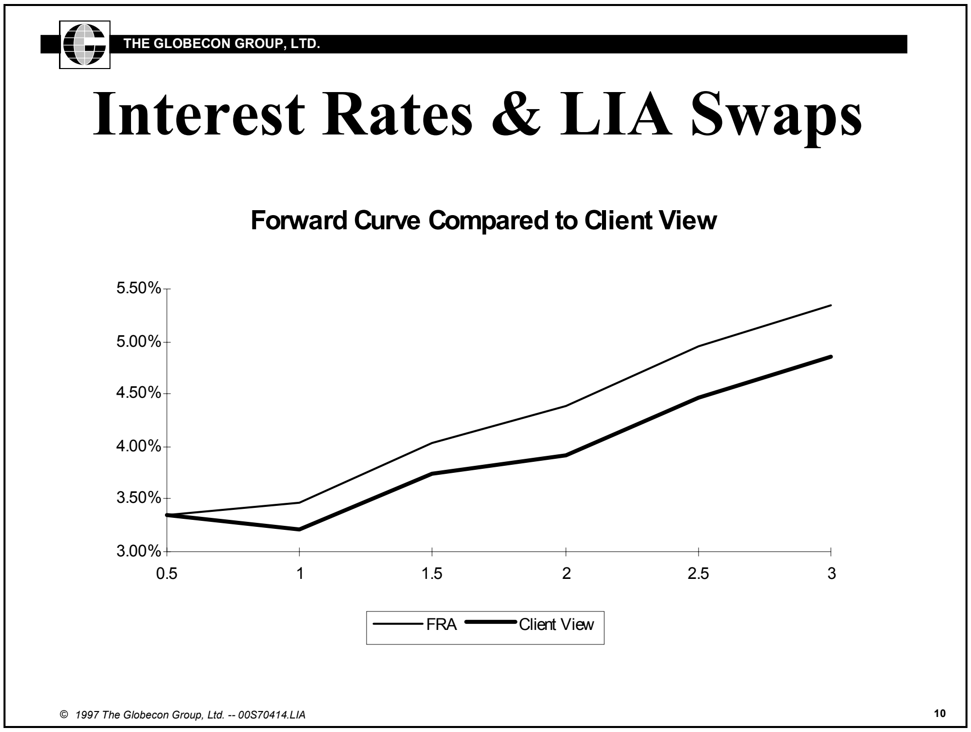
The swap curve remains upward sloping, with rates across the medium-term end of the curve rising steadily.

Remember that a LIA swap will have a fixed rate more or less equal to the forward swap rate one LIBOR reset period forward.

- The rate on a three-year LIA swap with six-month LIBOR resets would be about 4.58%.
- The rate using 12-month LIBOR resets would be about 5.157%.

The forward curve is very steeply upward sloping.

This suggests using one of the variations discussed above for clients which do not believe the short end of the curve will rise as quickly as the yield curve forecasts.

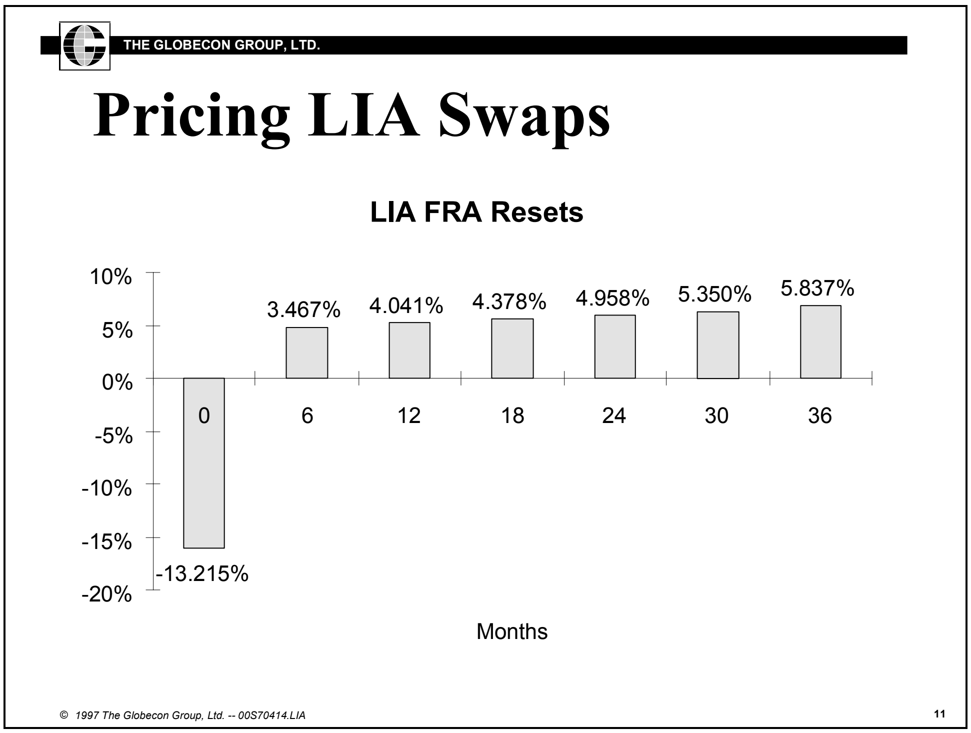


Let clients pay six-month LIBOR set in arrears and receive six-month LIBOR plus a spread of roughly 0.486% for three years.

PRICING LIA SWAPS

Pricing a LIA swap is rather like pricing a regular interest rate swap, except that the mark-to-market values for LIBOR are taken from the period **beginning** at each payment date, rather than ending at each payment date.

To price the three-year LIA swap, for example, use the strip of six-month FRAs beginning with the 6×12 FRA, but applying it to the 0×6 period:



The graph above uses the following rates:

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Pricing LIA Swaps

Date	*Days*	*Swaps*	*PV Factors*	*6-mo LIA FRA*	*FRA CF PV*
3/17/97			**1.000000**		**13.2150%**
9/17/97	**184**		**0.983564**	**3.467%**	**1.7431%**
3/17/98	**181**	**3.387%**	**0.966803**	**4.041%**	**1.9641%**
9/17/98	**184**		**0.947657**	**4.378%**	**2.1204%**
3/17/99	**181**	**3.835%**	**0.927359**	**4.958%**	**2.3118%**
9/17/99	**184**		**0.904925**	**5.350%**	**2.4747%**
3/17/00	**182**	**4.275%**	**0.881347**	**5.837%**	**2.6010%**

12

The present value of the FRA strip is calculated in the usual way, using actual days over 360 and the respective PV factors, according to the following formula, where *s* refers to the strip of semiannual periods. The only difference is that the formula uses the number of days from the period just ending instead of the number of days in the period just beginning:

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$$\text{FRA PV} = \sum_{s=1}^{n} \left(\text{FRA}_s \times \frac{\text{Days}_s}{360} \times \text{PVf}_s \right)$$

 13

For example, the PV for the first six-month period is 1.743%:

$$1.743\% = 3.467\% \times \frac{184}{360} \times 0.9836$$

In this case, on the fixed-rate side of the swap use the actual number of workdays on a 30/360 basis. The day count for the annual fixed-rate payments has to take into account the number of 30/360 days in each annual period.

Now solve for a single fixed-rate payment which, when adjusted for the number of 30/360 days from year to year, returns the same PV as the strip of FRAs above.

$$PV = \sum_{t=1}^{n}\left(Pmnt \times \frac{30/360\ Days_t}{360} \times PVf_t\right)$$

Since Pmnt is the same every period, it is possible to solve for it directly:

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$$Pmnt = \frac{PV}{\sum_{t=1}^{n}\left(PVf_t \times \frac{30/360\ Days_t}{360}\right)}$$

 14

In other words, the LIA swap fixed-rate payment is equal to the PV of the floating-rate cash flows divided by the sum of the relevant annual PV factors at 12 months, 24 months, and 36 months.

Calculate the rate:

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$$\text{Pmnt} = \frac{13.215\%}{(0.9668 \times \frac{360}{360} + 0.9273 \times \frac{360}{360} + 0.8813 \times \frac{360}{360})}$$

Pmnt = 4.761%

 15

This rate is close to the rate on a three-year swap beginning in six months, 4.583%.

It is some 0.486% higher than the rate on the normal three-year, which is 4.275%. This is the source of the LIA swap spread mentioned above.

HEDGING THE LIA SWAP

Hedging this swap using FRAs or other swaps requires consideration of LIBOR volatility.

This is because the FRAs are being used one period early.

The FRA values can be hedged, but the discount rates that will be used to move the FRA cash flows from the end of the period to the beginning cannot.

Thus, the future value of the LIBOR cash flows can be locked in, but not their "settlement values."

This will be based on the actual level of LIBOR, which is, of course, unknown.

But a range can be estimated using the FRAs and market volatility.

In this case, assuming a level of volatility for each FRA of 20% (probably a little high), adjust the expected future LIBOR levels as shown on the following page:

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Hedging LIA Swaps

Period	*LIA FRA*	*Lower Limit*	*Upper Limit*
0×6	**3.467%**	**2.975%**	**3.960%**
6×12	**4.041%**	**3.471%**	**4.610%**
12×18	**4.378%**	**3.756%**	**4.999%**
18×24	**4.958%**	**4.260%**	**5.657%**
24×30	**5.350%**	**4.591%**	**6.110%**
30×36	**5.837%**	**5.013%**	**6.662%**

16

In each case, the lower and upper limits are calculated using the following

relationships: $\sigma = \frac{1}{2} \times \frac{1}{\sqrt{\text{time}}} \times \ln\left(\frac{\text{Upper Limit}}{\text{Lower Limit}}\right)$

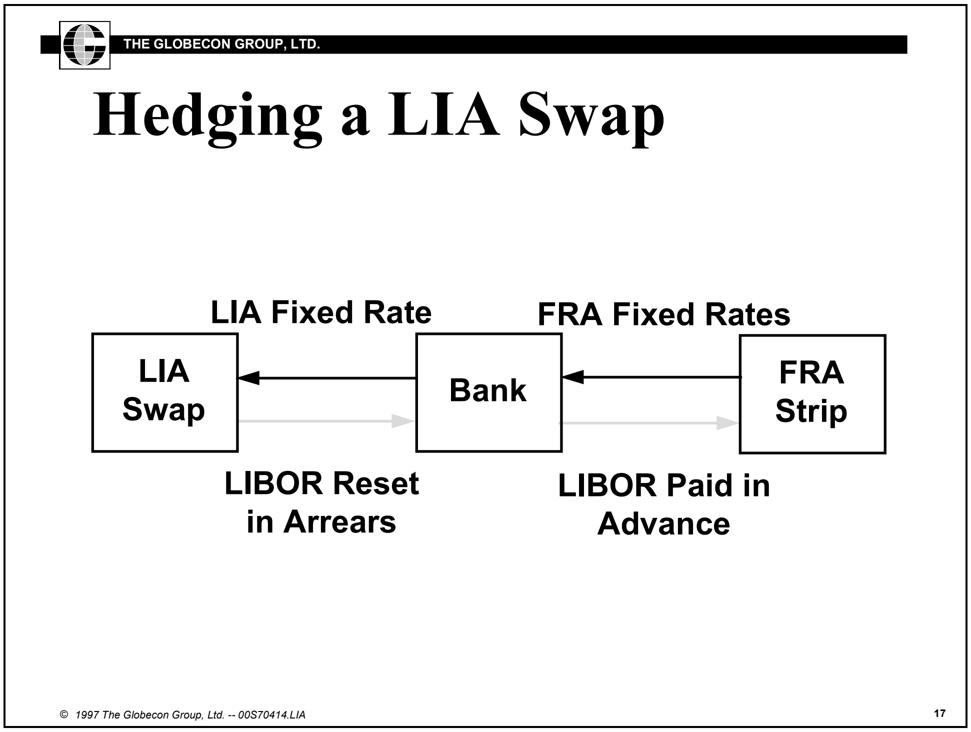
$$\text{FRA} = \text{Lower Limit} \times 50\% + \text{Upper Limit} \times 50\%$$

Combining these two equations leads to the following:

$$\text{Lower Limit} = \frac{2 \times \text{FRA}}{1 + e^{\left(2 \times \sigma \times \sqrt{t}\right)}}$$

$$\text{Upper Limit} = \text{Lower Limit} \times e^{\left(2 \times \sigma \times \sqrt{t}\right)}$$

Imagine the bank has taken the following position in a LIA swap:



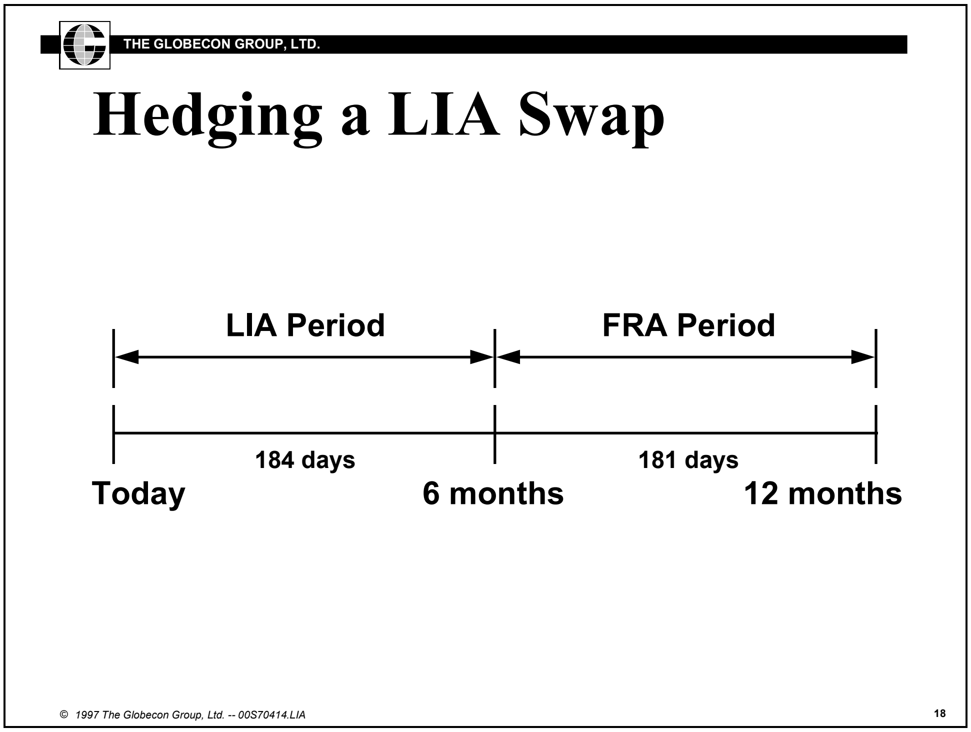
The strip of FRAs is a way to lock in the value of the LIBOR payments the bank will receive.

Selling the FRA for the LIA period beginning at the payment date provides a hedge of the rate used for the LIA payment.

The bank will receive a payment from the LIA swap based on the rate of LIBOR for the following six months. This is the same rate the FRA will refer to.

The LIA swap payment will take place at the beginning of the period. The bank will settle the FRA at the beginning of each period, too, so it will have about the cash flow it needs, but the FRA settlement will be for the PV of the cash flow at the beginning of the period.

These relationships can be depicted on the following time line for the very first six-month period in the deal, the 0×6 period:



The floating-rate payment received from the LIA swap will use the market level for six-month LIBOR applied to the first six-month period with 184 days:

$$\text{LIA Payment} = \text{LIBOR} \times \frac{184}{360} \times \text{Swap Notional Principal}$$

LIA Swaps and "Quanto Risk"

The FRA net payment will also use the market level of six-month LIBOR in six months. But it will apply it to the 6×12 period of 181 days and discount it to the beginning of the period at six months as follows:

$$\text{FRA Payment} = \frac{\text{LIBOR} \times \frac{181}{360}}{\left(1 + \text{LIBOR} \times \frac{181}{360}\right)} \times \text{FRA Notional Principal}$$

To make these two payments more or less equal, raise the notional principal on the FRA by a factor equal to the discount and adjust for the different number of actual days:

$$\text{FRA Payment} = \text{LIA Payment}$$

$$NP_{FRA} \times \frac{L \times \frac{181}{360}}{\left(1 + L \times \frac{181}{360}\right)} = NP_{Swap} \times L \times \frac{184}{360}$$

$$NP_{FRA} = NP_{Swap} \times \frac{184}{181} \times \left(1 + L \times \frac{181}{360}\right)$$

This adjustment will be different for each period, and will change as LIBOR changes. This makes it difficult to calculate the amount of adjustment needed in advance.

While it is possible to fix the FRA rate against LIBOR each period, it is not possible to fix the market rate at which the payment will be discounted to the beginning of each period. This adds a second degree of uncertainty into the hedge.

This uncertainty is a form of ***quanto risk***, or risk that cannot be hedged in advance.

It is also reasonable to assume that the discount rate will fluctuate from the lower to the upper limits calculated above, and calculate an adjustment to the LIA swap rate that keeps the bank whole when the FRA discount rates change to the extremes of one standard deviation.

In this case the adjusted rate is less than two basis points different. The effect is minimal.

This is well known among FRA traders using futures contracts to hedge their FRA positions.

It results from the fact that FRAs have convexity while futures contracts are linear.

FRA convexity results from the fact that the payout on the FRA is a function of changing LIBOR rates, which result in a changing discount rate.

Futures variation margin is not discounted from the end of the period to the beginning, but paid daily over the life of the contract.

Futures would thus represent a more interesting hedge, if there existed liquid futures contracts covering the full term of the LIA swap.

LIBOR-in-Arrears Swaps

This module covers LIBOR-in-arrears swaps: applications, pricing, and hedging. It can be used in any slightly more advanced FRM or fixed income workshop. A working knowledge of PV factors and basic swap pricing is necessary.

This module is a natural follow-up to basic swap pricing and logic, and can be used as part of an overall discussion of "advanced" swaps or variations of interest rate swaps.

The module should take approximately 30 minutes to cover.

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